19. J. J. Martin, W. McCabe, and C. C. Monrad, Chem. Eng. Prog., 47, 91-94 (1951).
20. N. N. Suntsov, S. L. Demenok, and V. V. Medvedev, "Hydraulic resistance of regular spherical packings," Submitted to VINITI 14.12.88, No. 8766-B 88, Moscow (1988).

WAVE REGIME OF CONSOLIDATION OF A POROUS COMPRESSIBLE MEDIUM

N. N. Zhilyaeva and A. M. Stolin

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An analytical solution in the form of a compression wave is found to the problem of the consolidation of a porous medium. Questions relating to the validity of the solution are examined.

In the study of the problem of the consolidation of a viscous compressible medium in the theory of hot pressing [1, 2], it is customary to ignore the inertial and nonsteady terms in the equations of motion and to replace these equations by simpler conditions of equilibrium [3-5]. This simplication is usually connected with small Reynolds numbers Re. The smallness of Re for the hot pressing of hard alloys is based on approximate calculations [4]. In this case, the initial variation of density in the material is important only in regard to the scale factor and has no effect on the character of the dependence of density on time. The perturbation from the piston is transmitted instantaneously to all discrete volumes of material. Such a consolidation regime has been called the regular regime [5].

Strictly speaking, the validity of ignoring inertial and nonsteady terms in the equations of motion depends not only on the smallness of Re, but also on the value of the partial derivatives of velocity with respect to the coordinates and time. At the same time, the inertia of the medium itself accounts for several fundamental characteristics of the process. It is important that the perturbation from the piston is not transmitted instantaneously to all discrete volumes in such media. Because this is the case, the preconditions are established for the formation of a compression wave in the porous medium. In connection with this, it is interesting to examine the problem of the compression of a porous medium with allowance for its inertia. In the present investigation, we seek to study the possibility of the occurrence of consolidation regimes other than the regular regime by solving the problem in the form of a compression wave. Here, we make use of the concept of intermediate asymptotes [6]. The solution of the problem of the compression of a porous medium with allowance for inertial and nonsteady terms allowed us to find the necessary conditions for occurrence of the regular consolidation regime — the conditions under which we can ignore the inertia of the medium. It is shown that the realization of both transitional and wave regimes of consolidation is possible. Distributions of density, velocity, and stress are found for materials which undergo consolidation in the wave regime.

Formulation of the Problem. We will examine the axial compression of a viscous porous medium under the influence of a piston moving from right to left. The motion of the medium during its consolidation is described by the equations of continuity and motion together with rheological relations and boundary conditions

$$\partial\rho/\partial t + \partial(\rho U)/\partial x = 0, \tag{1}$$

$$\rho\rho_1(\partial U/\partial t + U\partial U/\partial x) = \partial\sigma/\partial x, \tag{2}$$

$$\sigma = (4\eta/3 + \mu)\,\partial U/\partial x = \sigma_0\frac{\rho^m}{1-\rho}\,\partial U/\partial x, \tag{3}$$

Institute of Structural Macrokinetics, Academy of Sciences of the USSR, Chernogolovka.
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$\sigma_0 = {}^4/_3\eta_1$. Here, we take the following dependences for shear and bulk viscowity [3-5]: $\eta=\eta_1\rho^m$, $\mu=4\eta(\rho)\rho/3(1-\rho)$. We will assume that the compressible medium is infinite and we will assign boundary conditions at x = ±∞. If the length of the product is sufficiently great, then such an approximation is valid.

Let the material be stationary at x = -∞ and let us assume that it does not undergo consolidation here:

$$\rho(-\infty) = \rho_0;\ U(-\infty) = 0. \tag{4}$$

At x = +∞, we adopt the following conditions at the piston:

$$\rho(+\infty) = 1;\ U(+\infty) = U_p \tag{5}$$

for a regime with a specified velocity,

$$\rho(+\infty) = 1,\ \sigma(+\infty) = N \tag{6}$$

for a regime with a specified force.

We will assume the existence of a compression wave moving through the material at a constant velocity c = const. By a wave solution to the problem, we mean a solution of the form f(x − ct). Let us change over to a moving coordinate system ξ, τ connected with the travelling wave:

$$\xi = x - ct,\ \tau = t. \tag{7}$$

In the new coordinates, the steady-state process of propagation of the compression wave is described by the system of equations

$$\partial[\rho(U-c)]/\partial\xi = 0, \tag{8}$$

$$\rho\rho_1(U-c)\,\partial U/\partial\xi = \partial\sigma/\partial\xi, \tag{9}$$

$$\sigma = \sigma_0 \frac{\rho^m}{1-\rho}\,\partial U/\partial\xi \tag{10}$$

with the boundary conditions

$$\xi = -\infty,\ \rho = \rho_0,\ U = 0; \tag{11}$$

$$\xi = +\infty,\ \rho = 1,\ U = U_p. \tag{12}$$

For the regime with the specified force, instead of (12) we have

$$\xi = +\infty,\ \rho = 1,\ \sigma = N. \tag{13}$$

Regime with Specified Piston Velocity. In this regime, U_p = const. In this case, by integrating (8) with allowance for the boundary conditions, we find the velocity of the compression wave:

$$c = U_p/(1-\rho_0), \tag{14}$$

i.e. the velocity of this wave is independent of the viscosity of the medium and is determined only by the velocity of the piston and the initial density of the material. It can be seen from (14) that the compression wave always precedes the piston, "running away" from it.

We now introduce the quantity

$$j = \rho (U - c), \tag{15}$$

which is the flux in the moving coordinate system. Integrating (8), we find that the flux is the same at all points of the material:

$$j = \rho (U - c) = U_p - c = - \rho_0 c = \text{const.} \tag{16}$$

Equation (16) can be rewritten as follows:

$$\Delta\rho/\rho = U/c, \tag{17}$$

which is evidence of similitude relative to the change in density and the velocity field in the material.

As a result of the integration of (9), we obtain

$$\sigma = \rho_1 j U. \tag{18}$$

Using rheological relation (10), we find it easy to obtain the following implicit expression for density $\rho(\xi)$ from (18):

$$\int\limits_{\rho_*}^{\rho} [\rho^{m-1}/(\rho - \rho_0)(1 - \rho)]\, d\rho = - \rho_1 c \xi/\sigma_0, \tag{19}$$

where $\rho_* = \rho(\xi = 0)$. Quantity ρ_* can take arbitrary values on the interval $(\rho_0; 1)$ due to the invariance of the wave solutions with respect to shear along the coordinate ξ.

We will henceforth use a linear dependence of shear viscosity on density (m = 1). Then from (19) we obtain the density profile

$$\rho (\xi) = (\rho_0 + a \exp (\mathrm{Re}\,\bar{\xi}))/(1 + a \exp (\mathrm{Re}\,\bar{\xi})) \tag{20}$$

and the corresponding profiles of velocity and stress

$$U (\xi) = c + j/\rho = \frac{U_p a \exp (\mathrm{Re}\,\bar{\xi})}{\rho_0 + a \exp (\mathrm{Re}\,\bar{\xi})}, \tag{21}$$

$$\sigma (\xi) = \sigma_0 U_p a \rho_0 A \exp (\mathrm{Re}\,\bar{\xi})/(1 - \rho_0) (a + \exp (\mathrm{Re}\,\bar{\xi}). \tag{22}$$

Here, we have used the notation $a = (\rho_* - \rho_0)/(1 - \rho_*)$, $A = \rho_1 c (1 - \rho_0)/\sigma_0$, $\bar{\xi} = \xi/H_0$, where H_0 is length of the product. At $\xi = +\infty$, we can use (22) to determine the force on the piston

$$\sigma (+\infty) = N = - \rho_0 \rho_1 U_p^2/(1 - \rho_0). \tag{23}$$

Regime with Specified Force on the Piston. In this case, the problem reduces to determination of piston velocity from (18):

$$U_p = \sqrt{|N| (1 - \sigma_0)/\rho_0 \rho_1}. \tag{24}$$

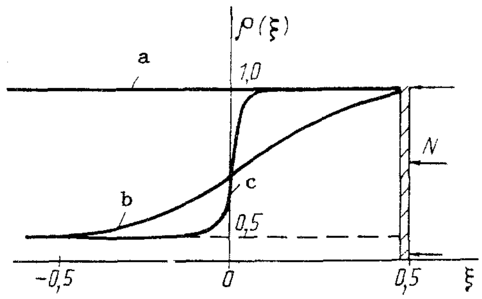

Fig. 1. Distribution of density over the coordinate ξ, calculated for a semifinished product 1 m long: a) with $\eta_1 = 10^{10}$ Pa·sec; b) 10^5; c) 10^4. ρ is dimensionless; ξ, m.

Here, the expressions for the distributions of density, velocity, and stress in the material coincide with those found earlier for the case of an assigned velocity (20-22).

For a medium with a constant viscosity, i.e., in the case when rheological relation (3) has the form

$$\sigma = k\partial U/\partial\xi \ (k = \text{const}), \tag{25}$$

the solution of system (8-13)

$$U = \exp[j\rho_1\xi/k - B] \tag{26}$$

does not satisfy the boundary condition on the piston (B is the constant of integration). The lack of a wave solution to the problem in this case means that the compression wave owes its existence to the compressibility properties of the material — more specifically, to the dependence of bulk viscosity on density.

One issue that remains is to determine the range of application of the wave solution that has been found. One obvious prerequisite for the realization of a compression wave is that the characteristic dimension of the wave be small compared to the size of the material, i.e.,

$$\delta \ll H_0, \tag{27}$$

where δ is the width of the wavefront (the length of the interval along the ξ axis in which density changes from $\rho_0 + \varepsilon$ to $1 - \varepsilon$, where ε is a small number). Using the expression for density (20), we can calculate the width of the wavefront for fixed values of piston velocity and the density and viscosity of the incompressible base:

$$\delta = -H_0 \ln \varepsilon^2/\text{Re} = -\sigma_0 \ln \varepsilon^2/\rho_1 U \ . \tag{28}$$

It is evident from (28) that the density profiles differ in relation to the value of Re. Figure 1 shows profiles of density calculated from (20) with the following parameter values: $\rho_0 = 0.5$; $\rho_* = 0.75$; $\rho_1 = 5\cdot10^3$ kg/m³; $U_p = 10^2$ m/sec; $H_0 = 1$ m; a = 1; $\varepsilon = 0.01$. Curves a, b, and c correspond to different values of viscosity of incompressible base η_1. At $\eta_1 = 10^{10}$ Pa·sec, the width of the compression wave $\delta \sim 10^5$ m $\gg H_0$ (Fig. 1a), i.e., condition (27) is violated. In this case, the solution found for the problem in [5] in Lagrangian coordinates is a good approximation of the actual solution and provides evidence of consolidation of the material in the regular regime. At $\eta_1 = 10^4$ Pa·sec (Fig. 1c), condition (27) is satisfied: $\delta \sim 10^{-1}$ m $\ll H_0$. The density profile that is constructed corresponds to the wave regime of consolidation. If $\eta_1 = 10^5$ Pa·sec, then the width of the consolidation front is comparable to the dimensions of the product: $\delta \sim 1$ m $\sim H_0$ (Fig. 1b), i.e., the consolidation front is "blurred." Thus, for the above-chosen parameters in the region of viscosity values $5\cdot10^4$ <

$\eta_1 < 10^6$, the indications are that neither the wave regime ($\delta << H_0$) nor the regular regime ($\delta >> H_0$) exist, and Eqs. (20-22) fail to descibe the consolidation process. We can use the term "transitional" to describe this consolidation regime — in which the compression wave becomes degenerate in the sense that velocity is not a constant value and the front itself erodes. This regime combines properties of both the wave regime and the regular regime. The boundaries of the transitional regime are to a certain extent conditional: the regime occurs where no regular or wave regime exists. In studies of the possibility of the occurrence of the wave regime in a material, consideration should be given to the final dimensions of the product and the limited pressing time. This is because the time and region of formation of the compression wave may in practice be so great that there is not sufficient time for the wave regime to materialize. It should be noted that the pressing time which is optimum for the consolidation of a product is determined as follows:

$$\tau_p^* = H_0/c. \tag{29}$$

In the case of incomplete consolidation of the product, it is possible to determine the ratio of the consolidated portion to the unconsolidated portion. This ratio is equal to the ratio of the time of pressure application to the optimum pressing time: τ_p/τ_p^*.

Here, we have only presented the necessary condition for the realization of wave regime (27). Determination of the sufficient conditions requires the solution of the problem in the most general form: with allowance for the finiteness of the product and the transience of the consolidation process itself. If the problem were formulated in this manner, it would be possible to clearly indicate the boundaries separating one regime from the other, determine the properties of the transitional regime, and examine questions dealing with the formation of the consolidation front.

NOTATION

t, x and τ, ξ, time and running length in the stationary and moving coordinate systems, respectively; σ, U, stresses and velocities in the material; η, μ, shear and bulk viscosity of the material; ρ_1, η_1, density and viscosity of the incompressible base; ρ_0, ρ, initial and running density of the material, referred to the density of its incompressible base; U_p, N, velocity and force on the piston; c, rate of propagation of compression wave in the material; H_0, size (length) of the material before consolidation; τ_p, time of application of pressure (pressing time).

LITERATURE CITED

1. V. V. Skorokhod, Rheological Foundations of the Theory of Sintering [in Russian], Kiev (1972).
2. M. S. Koval'chenko, Theoretical Principles of the Hot Shaping of Porous Materials [in Russian], Kiev (1980).
3. L. M. Buchatskii, A. M. Stolin, and S. I. Khudyaev, "Consolidation of a powdered material with a nonuniform distribution of density for different hot-pressing regimes," Preprint OIKhF AN SSSR, Chernogolovka (1986).
4. L. M. Buchatskii, A. M. Stolin, and S. I. Khudyaev, Poroshk. Metall., No. 12, 9-14 (1987).
5. A. M. Stolin, N. N. Zhilyaeva, and B. M. Khusid, Inzh.-Fiz. Zh., 59, No. 2, 248-254 (1990).
6. G. I. Barenblatt and Ya. B. Zel'dovich, Usp. Mat. Nauk, 2, No. 26, 115-129 (1973).